Name: **Version #1**

Instructor: Annette McP

Math 10120 Exam 1
Sept. 16, 2021.

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators **are** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 11 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)
10.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice _____

11. _____

12. _____

13. _____

14. _____

15. _____

Total _____

Name: _____

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2	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
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10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice _____

11. _____

12. _____

13. _____

14. _____

15. _____

Total _____

 Initials: Ap.

Multiple Choice

1.(5pts) Let

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$
$$A = \{2, 3, 4, 5, 6\}$$
$$B = \{2, 4, 6, 8, 10\}.$$
$$C = \{4, 5, 6, 7, 8\}.$$

$B' = \{1, 3, 5, 7, 9\}$

Which of the following sets is equal to $(A \cap B') \cup C$?

(a) $\{3, 4, 5, 6, 7, 8\}$ (b) $\{4, 5, 6, 7\}$ (c) $\{4, 5, 6, 7, 9\}$

(d) $\{3, 5\}$ (e) $\{5\}$

$A \cap B' = \{3, 5\}$

$(A \cap B') \cup C = \{3, 4, 5, 6, 7, 8\}$

2.(5pts) Consider the following sets

U = {English Words}

A = {Five letter English words}

B = {English words with at least three different (see below) vowels}

C = {English words starting with the letter "p"}

Which one of the following words is in the set

$$(A' \cup B) \cap C$$

[**Note:** two vowels are different if they are different letters of the alphabet. For example the word "elevate" is not in the set B but the word "elevator" is in the set B]

(a) pony (b) petal (c) print

(d) vaccinate (e) eunoia

A word in $(A' \cup B) \cap C$ must be in C (starts with p) and either A' or B i.e (not have 5 letters OR at least 3 different vowels).

Pony starts with p and does not have 5 letters, so it is in A' and in C.

3.(5pts) In a survey of 80 music students, 35 played piano, 40 played the violin and 10 played neither the piano nor the violin. How many of the music students played both instruments?

(a) 5 (b) 10 (c) 15 (d) 30 (e) 0

$n(U) = 80$

$n(P) = 35$

$n(V) = 40$

$n((P \cup V)') = 10 \Rightarrow \boxed{n(P \cup V) = n(U) - n((P \cup V)')}$

$= 80 - 10 = \boxed{70}$

Q: $n(P \cap V) = ?$

$n(P \cup V) = n(P) + n(V) - n(P \cap V)$

$70 = 35 + 40 - n(P \cap V) \Rightarrow \boxed{n(P \cap V) = 5}$

4.(5pts) Liam is competing in a race. From the starting line, he must take a canoe, a kayak, a paddleboat or a rowboat to point A. From point A to point B he must ride a horse or a bicycle, and from point B to the finish line, he must skateboard, rollerblade, or ride a scooter. In how many ways can Liam complete the race?

(a) 24 (b) 18 (c) 9 (d) 3 (e) 8

Mult. Principle

Step 1 · Step 2 · Step 3

START → A · A → B · B → Finish

4 ways · 2 ways · 3 ways

ways to complete Race

$= 4 \cdot 2 \cdot 3 = \boxed{24}$

 Initials: _____

5.(5pts) How many different words, including nonsense words can be made by rearranging the letters of the word

W A T T A M O L L A

Some Letters Same.

(a) $\frac{10!}{3! \cdot 2! \cdot 2!}$ (b) $10!$ (c) $\frac{6!}{3! \cdot 2! \cdot 2!}$

(d) $\frac{10!}{7!}$ (e) $\frac{6!}{3!}$

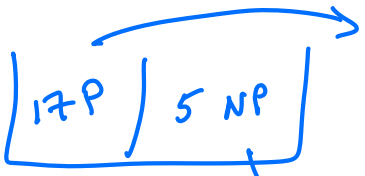
W → 1
A → 3
T → 2
M → 1
O → 1
L → 2
10

$$\frac{10!}{1!\,3!\,2!\,1!\,1!\,2!} = \frac{10!}{3!\,2!\,2!}$$

6.(5pts) The Hurling Club of Kilclonfert has 17 players and 5 non-players. The non-players are qualified to serve as linesmen and the players are not. In how many ways can the club choose a team of 15 players and two linesmen (from the 5 qualified to do the job) for their next game, if the order in which the team members and the linesmen are selected does not matter?

(a) $17 \cdot 16 \cdot 5$ (b) $C(22, 17)$ (c) $10 \cdot \frac{17!}{15!}$

(d) $17^{15} \cdot 5^2$ (e) $10 \cdot \frac{17!}{2!}$



step 1 Choose 15 players
C(17, 15) ways

step 2 choose 2 Linesmen
C(5,2) way

ways = C(17, 15) · C(5, 2)
= C(17, 2) C(5, 2) = $\frac{17 \cdot 16}{2 \cdot 1} \cdot \frac{5 \cdot 4}{2 \cdot 1}$ = 17.16.5

 Initials: _____

7.(5pts) Nadxielli wants to fill out a bracket for her dorm pool, for a knockout basketball tournament in which 64 teams will play. She needs to choose teams for first, second, third and fourth place. Ties are not allowed in the tournament and at the time Nadxielli fills out her bracket all teams have a chance of winning any of the 4 top places. In how many ways can Nadxielli fill out her bracket?

(a) $P(64,4)$ (b) $C(64,4)$ (c) $64!$ (d) $\frac{64!}{4!}$ (e) $4!$

$\frac{64}{1^{st}\text{ place}} \cdot \frac{63}{2^{nd}\text{ pl}} \cdot \frac{62}{3^{rd}\text{ pl}} \cdot \frac{61}{4^{th}\text{ pl.}} = P(64,4)$

ORDER MATTERS

8.(5pts) How many 4-digit numbers (the numbers cannot start with a 0) have no repeating digits?

(a) $4,536$ (b) $5,040$ (c) $10,000$ (d) $9,000$ (e) $3,024$

zero not allowed ↓ Have 9 choices (0 is allowed) ↙

$\frac{9}{\text{Dig 1}} \cdot \frac{9}{\text{Dig 2}} \cdot \frac{8}{\text{Dig 3}} \cdot \frac{7}{\text{Dig 4.}}$

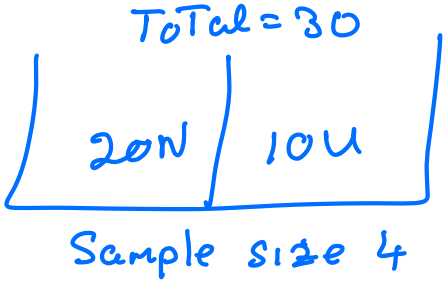
$= 4536$

9.(5pts) A box contains 30 AAA batteries. Ten of the batteries are used and 20 are new. How many samples of 4 batteries from the box contain at least one used battery?

(a) $C(30,4) - C(20,4)$ (b) $10 \cdot C(10,2) \cdot C(10,3) \cdot C(10,4)$

(c) $10 + C(10,2) + C(10,3) + C(10,4)$ (d) 10

(e) $C(30,4) - C(10,4)$



samples with at least one used battery

= Total # samples − # samples with 0 used batteries

= $C(30,4) - C(20,4)$

all N batteries = 0 used.

10.(5pts) Spencer is about to roll a six sided die 4 times. the sides are labelled 1 through 6. Spencer will write down the sequence of numbers he gets in the order in which they occur. How many of the possible sequences that could result from Spencer's experiment have exactly two sixes.

(a) $C(4,2) \cdot 25$ (b) 25 (c) 6^4

(d) $P(4,2) \cdot 25$ (e) $6^4 - 25$

_ _ _ _

To create such a sequence:

step 1 choose 2 places for the two sixes $C(4,2)$ ways.

step 2 choose a number from 1–5 for first empty place 5 ways

Step 3 choose a num. from 1–5 for 2nd spot.

$C(4,2) \cdot 5 \cdot 5 = C(4,2) \cdot 25$ ways

7. Initials: _____

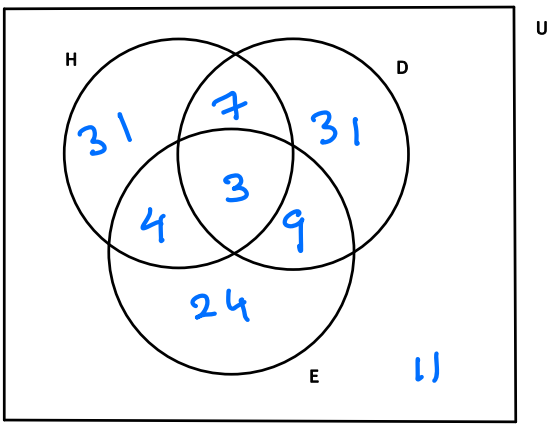
For Questions 12-14, you may express your answers using the notation for permutations, combinations, powers and factorials, where appropriate

11. (12pts) A survey of 120 visitors to the campus of The University of Notre Dame were asked which of three popular dining areas they had eaten at during their visit.
45 of them said they had eaten at The Huddle
50 of them said they had eaten at The Duncan Center,
and 40 of them said they had eaten at Eddy Street Commons.

Ten of those surveyed ate at The Huddle and The Duncan Center
7 of them ate at The Huddle and Eddy Street Commons,
and 12 of them ate at The Duncan Center and Eddy Street Commons.
Three of those surveyed ate at all three places.

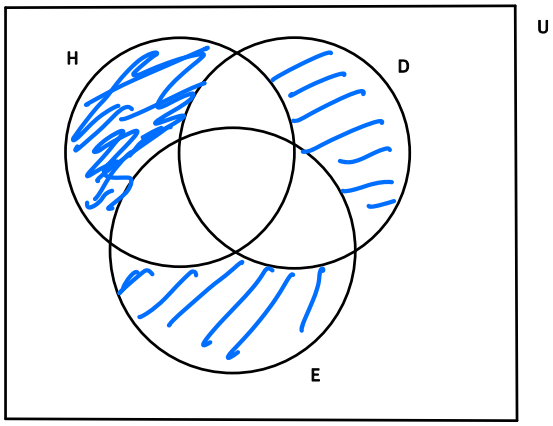
(a) Place the appropriate number of elements in each of the 8 basic regions in the diagram on the right.

#	
U	120
H	45
D	50
E	40
H∩D	10
H∩E	7
D∩E	12
H∩E∩D	3



H = Ate at The Huddle.
D = Ate at The Duncan Center.
E = Ate at Eddy St. Commons.

(b) Shade the region(s) in the diagram on the right which correspond(s) to visitors who ate at exactly one of the three dining areas.



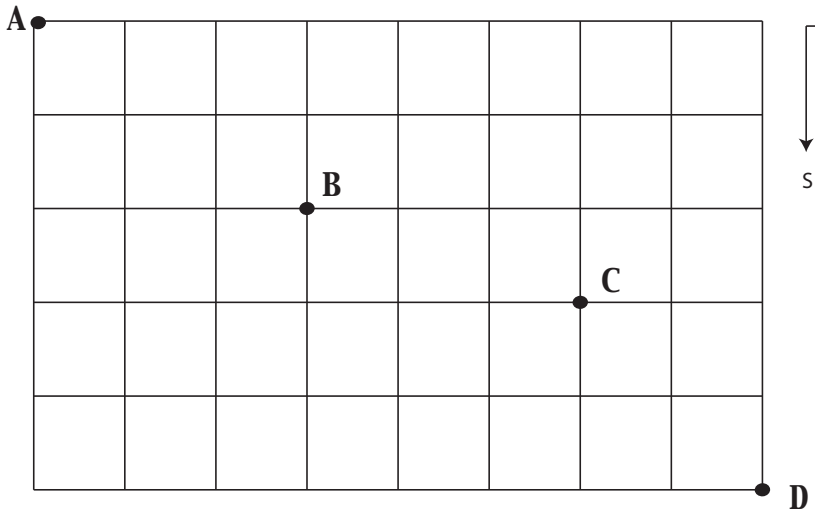
H = Ate at The Huddle.
D = Ate at The Duncan Center.
E = Ate at Eddy St. Commons.

(c) How many of the visitors surveyed did not eat at any of the three dining areas mentioned?

11

8. Initials: _____

12.(12pts) Alan is going to go for a jog through his neighborhood. Alan always jogs efficiently and chooses a route that does not backtrack (that is, he only ever jogs South or East).



8E
5S
= 13 Blocks

The questions below refer to the street map shown above.

(a) How many jogging routes can Alan take from his home **A** to that of his friend Daniel at **D**?

$\frac{13!}{5!\,8!} = C(13,8)$ = # words with 8 E's and 5 S's
a.k.a
sequences of S's and E's with 8 E's

(b) How many routes can Alan take if he intends to jog from his home **A** to that of his friend Daniel at **D**, and he intends to stop briefly at the coffee shop at **C**.

= # Routes A → D thru C
= step 1 · step 2
= # Routes A → C · # routes C → D
= $C(9,6) \cdot C(4,2)$

(c) How many routes can Alan take if he intends jog from his home **A** to his friend Daniel's house at **D**, stopping briefly at the coffee shop at **C**, but avoiding the intersection at **B**.

(b)
= # Routes A → D Thru C − # Routes A → D thru B and C

= $C(9,6)\cdot C(4,2) - C(5,2)\cdot C(4,3)\cdot C(4,2)$
$A\to B \cdot B\to C \cdot C\to D$

= $\left(C(9,6) - C(5,2)\cdot C(4,3)\right)\cdot C(4,2)$

 Initials: _____

13.(12pts) How many different words with five letters, including nonsense words, can be made from the letters of the word

C O R O N A C O A S T E R

if:

(a) Letters CANNOT be repeated.

The set of Letters we choose from is {C, O, R, N, A, S, T, E}. 8 Letters

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$

(b) Letters CANNOT be repeated and the word must start with an E.

$1 \cdot 7 \cdot 6 \cdot 5 \cdot 4$

(c) Letters CANNOT be repeated and the middle letter must be an E.

$7 \cdot 6 \cdot 1 \cdot 5 \cdot 4$

(d) Letters CANNOT be repeated and at least one letter must be an E.

Either 1st Letter is E ; $7 \cdot 6 \cdot 5 \cdot 4$ ways

+

OR

The second Letter is E ; $7 \cdot 6 \cdot 5 \cdot 4$ ways

+

OR

The middle Letter is E ; $7 \cdot 6 \cdot 5 \cdot 4$ ways

+

OR

The penultimate Letter is E ; $7 \cdot 6 \cdot 5 \cdot 4$ ways

+

OR

The Last Letter is E ; $7 \cdot 6 \cdot 5 \cdot 4$ ways

Ans $\boxed{4 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \text{ ways}}$

14.(12pts) A hand (sample) of 5 cards is drawn from a standard deck of 52 cards. Recall that a standard deck of cards has 13 denominations (2's, 3's, ..., 10's, Jacks, Queens , Kings, Aces) and 4 suits (Hearts, Diamonds, Spades, Clubs) and note that the order of the cards in a hand does not matter.

(a) How many different hands consist entirely of hearts?

13 hearts

$C(13, 5) =$

(b) How many different hands consist of 3 kings and 2 cards from a different denomination (the two cards come from the same denomination?)

$C(4,3)$ (choose 3 k's) $\cdot$ 12 (choose Diff. denomination) $\cdot$ $C(4,2)$ (choose 2 from that denom) $= 288$

(c) How many different hands consist of four cards from the same suit and one card from another suit?

4 (choose a suit) $\cdot$ $C(13,4)$ (choose 4 cards) $\cdot$ 3 (choose a diff suit) $\cdot$ 13 (choose card for it)

$= 111,540$

11. Initials: _____

15.(2pts) You will be awarded these two points if you write your name in CAPITALS and you mark your answers on the front page with an X through your answer choice like so: ~~(a)~~ (not an O around your answer choice) . You may also use this page for

ROUGH WORK